

# Advanced Biostatistics for Medical Researchers

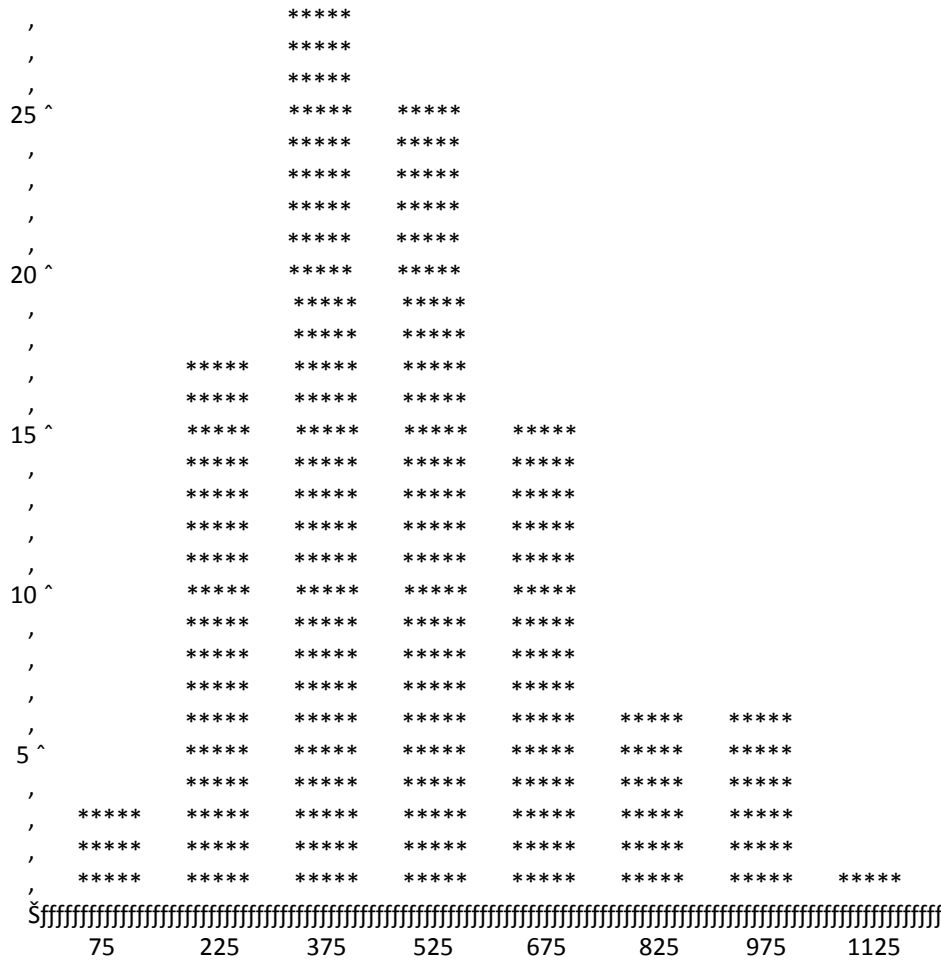
Longitudinal Analysis

# Approaches to Longitudinal Data Analysis

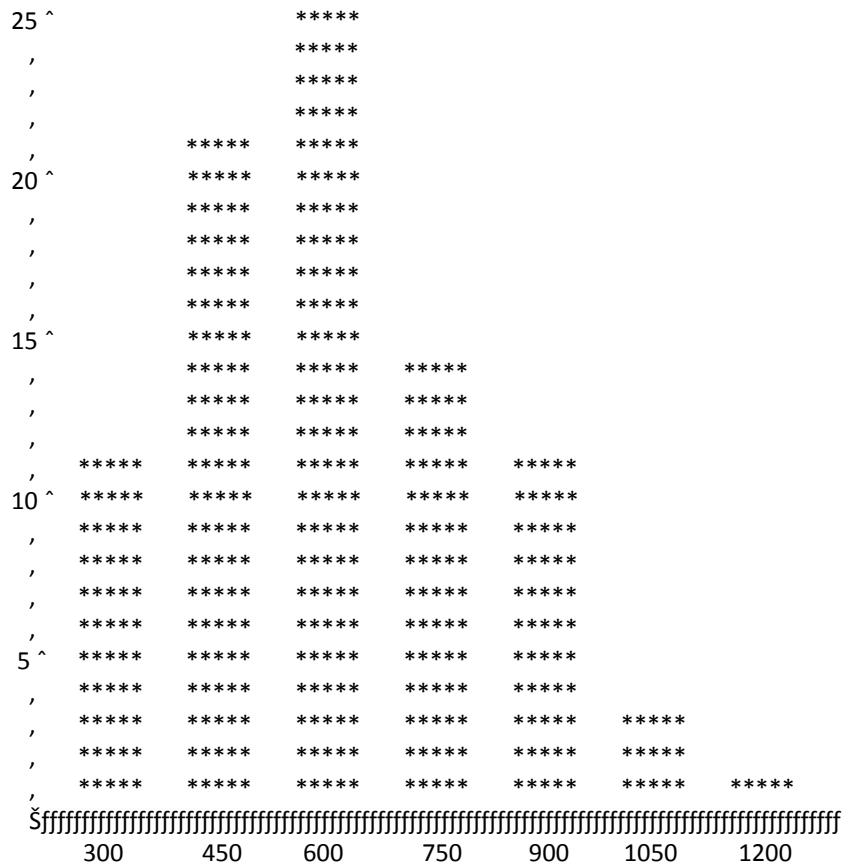
## References:

- “Analysis of Longitudinal Data”. PJ Diggle, K-Y Liang, SL Zeger. 1994. Oxford University Press (Difficult)
- “Statistical Methods for the Analysis of Repeated Measurement”. CS Davis. 2002. Springer-Verlag (Less Difficult)
- “Applied Longitudinal Analysis”. GM Fitzmaurice, NM Laird, JH Ware. 2004. John Wiley & Sons. (Best)

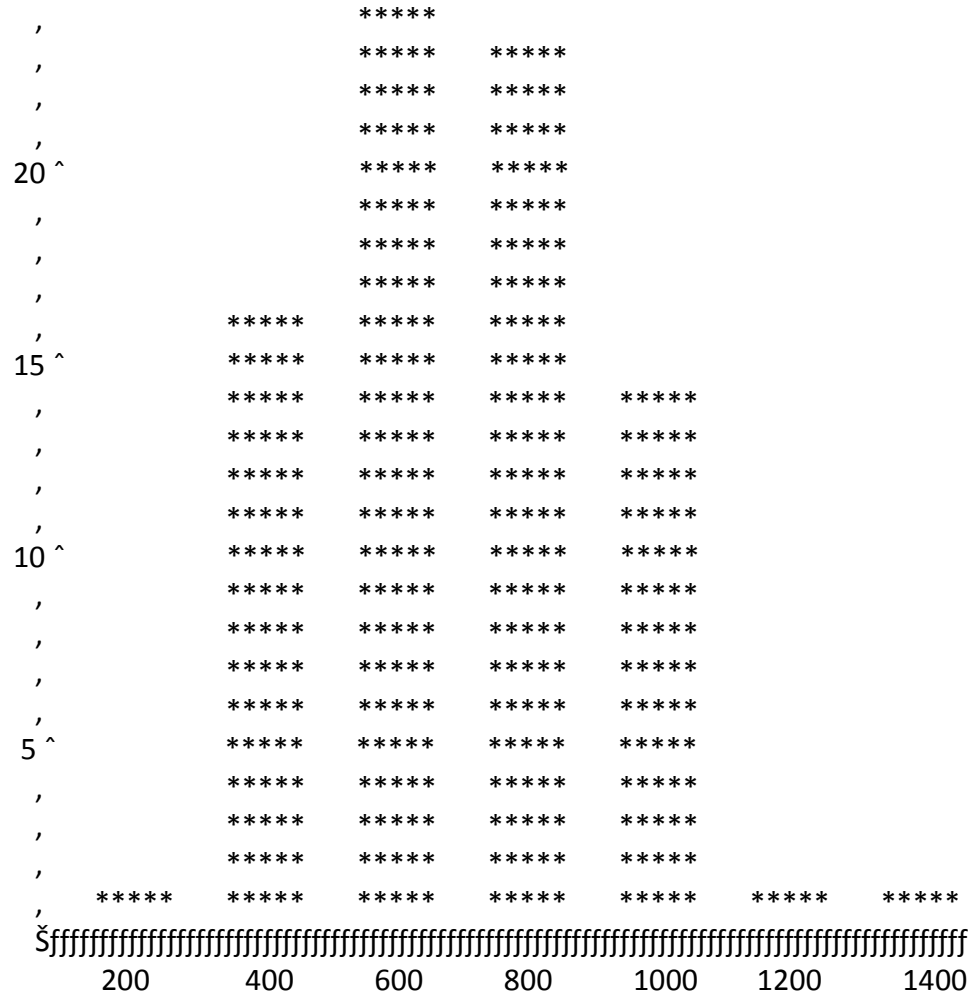
**Baseline CD4 Counts: n=101, Mean=487 / Median=463, Range: [42,1093]**



**6-Month CD4 Counts: n=86, Mean=620 / Median=606, Range: [249,1213]**



**12-Month CD4 Counts: n=80, Mean=695 / Median=683, Range: [168,1456]**





## Usual Approach for Independent Data: Linear Regression

$$CD4_i = \alpha + \beta \text{Time}_i + \varepsilon_i$$

where the subscript “i” goes across all of the measures of CD4 counts;

Assumption: The  $\varepsilon_i$  are independent of one another. → The CD4 counts are independent of one another.

### Result:

$$CD4 = 495 + 17.6 \times \text{Time (in months)} + \text{Error}$$

Where  $n = 267$

$$\text{Time effect} = 17.6; \quad \text{se} = 2.72; \quad p < .0001$$





Problem: In this data set, there are multiple measurements of CD4 counts on each of 101 subjects. (1 to 3 measurements per subject, taken at the start of treatment and then at 6 and 12 months)

→ We violate the independence assumption

More Appropriate Model:

$$CD4_{ij} = \alpha + \beta \text{Time}_{ij} + \varepsilon_{ij}$$

where “i” is an index for the 101 subjects (i.e., 1, 2, 3, ..., 101)  
and “j” is an index for serial measures on each subject  
(i.e., j goes from 1 to 1, 2, or 3, depending on the subject)

Assumptions:

a. Measures of CD4 from different subjects are independent of one another.

b. Serial measures of CD4 from the same subject are correlated with each other.

→ **Repeated Measures Linear Regression Model**

(as opposed to a **Random Effects Model** or a **Random Coefficients Model**)

## Comparisons Between Subjects (with each subject having multiple correlated outcomes)

Basic Idea: If the CD4 counts are correlated within subject and, say, we want to compare CD4 counts between HIV-infected and HIV-uninfected subject (or, between treated and untreated; etc), then

a. 2 measurements on the same subject are not worth as much as 1 measurement on each of 2 subjects

or

b. 2 measurements on the same subject are not worth twice as much as one measurement

For example,

If Correlation=0, then  $2 = 2 \times 1$

If Correlation=1, then  $2 = 1$

If  $0 < \text{Correlation} < 1$ , then  $2 = 1.?$

i.e., If Corr = .10, then  $2 = 1.8$

If Corr = .25, then  $2 = 1.6$

If Corr = .50, then  $2 = 1.3$

## Adjusting Sample Size Calculations For Correlated Data

Usual Situation: Common statistical tests and sample size calculations assume that the outcome measure acts independently from one measurement to the next. (If I told you the result of one measurement of the outcome, this would be totally uninformative about any other measurement of the outcome.)

This independence assumption is usually reasonable if the measurements are made only once on each subject, and the subjects have no relationship to each other.

This assumption is generally suspect in examples such as:

1. Each subject is measured at 4 time points.
2. The subjects are related (i.e., siblings or littermates)
3. Each subject contributes multiple measurements (i.e., digits, teeth, eyes, multiple segments of artery)

### Situation of Current Interest:

- Assume that the experiment involves 25 subjects, but each subject is measured 4 times.
- This means that there are  $4 \times 25 = 100$  outcome measurements available for analysis.
- However, the “effective sample size” is somewhere between  $N=25$  (the number of subjects) and  $N=100$  (the number of measurements).
- The true number depends on the magnitude of the correlation between the 4 measurements on each subject.

### Definition 1: (Intra-Class) Correlation Coefficient

$$\rho = \sigma^2_{\text{Between}} / (\sigma^2_{\text{Between}} + \sigma^2_{\text{Within}})$$

(This ranges from 0 to 1, where we always presume positive correlation.)

### Definition 2: Design Effect (D)

Assume that there are k correlated measures on each “subject”

$$D = 1 + (k-1) \rho$$

Result: Let  $N_{\text{Random}}$  represent the number of observations that would be required to carry out the study if all the observations were independent.

Then 
$$N_{\text{Random}} = N_{\text{Correlated}} / D$$

Prior Example: If we assume that for each of the 25 subjects, the correlation between the k=4 repeated measurements was  $\rho=0.5$ ,

Then 
$$D = 1 + 3(.5) = 2.5$$

$$N_{\text{Correlated}} = 100$$

And 
$$N_{\text{Random}} = N_{\text{Correlated}} / D = 100/2.5 = 40$$

So, the “effective sample size” is 40

## Example for Correlated Data

**Sample Size Required, Per Group**  
Power=90%, Type I Error=5% (2-sided)

**Standardized Effect Size = .50**

|                                   | <u>Number of<br/>Patients</u> | <u>Number of<br/>Measurements</u> |
|-----------------------------------|-------------------------------|-----------------------------------|
| <u>Independent Data:</u>          | 84                            | 84                                |
| <br>                              |                               |                                   |
| <u>2 Measurements Per Subject</u> |                               |                                   |
| $\rho = .05$                      | 44                            | 88                                |
| $\rho = .10$                      | 46                            | 92                                |
| $\rho = .25$                      | 52                            | 105                               |
| $\rho = .50$                      | 63                            | 126                               |
| <br>                              |                               |                                   |
| <u>3 Measurements Per Subject</u> |                               |                                   |
| $\rho = .05$                      | 31                            | 92                                |
| $\rho = .10$                      | 34                            | 101                               |
| $\rho = .25$                      | 42                            | 126                               |
| $\rho = .50$                      | 56                            | 168                               |
| <br>                              |                               |                                   |
| <u>5 Measurements Per Subject</u> |                               |                                   |
| $\rho = .05$                      | 20                            | 101                               |
| $\rho = .10$                      | 24                            | 118                               |
| $\rho = .25$                      | 34                            | 168                               |
| $\rho = .50$                      | 50                            | 252                               |

## Comparisons Within Subjects (with each subject having multiple correlated outcomes)

Basic Idea: If the CD4 counts are correlated within subject and, say, we want to compare how CD4 counts change from baseline to follow-up within each subject (or, we do a cross-over study; etc), then:

- a. 2 measurements on the same subject are worth **more** than 1 measurement on each of 2 subjects
- b. 2 measurements on the same subject are worth **more** than twice as much as 1 measurement

For example,

|                   |                       |
|-------------------|-----------------------|
| If Correlation=0, | then $2 = 2 \times 1$ |
| If Corr = .10,    | then $2 = 2.2$        |
| If Corr = .25,    | then $2 = 2.7$        |
| If Corr = .50,    | then $2 = 4.0$        |

As another example,

Assume we want to compare CD4 counts at diagnosis to CD4 counts 1-year after diagnosis, where CD4 counts have a standard deviation of 250 and change by 100 over the year.

- a. If we measure 100 subjects at the time of diagnosis and a different 100 subjects at 1 year, we will have 80% power.
- b. If we measure the same 100 subjects both at diagnosis and at 1 year, and assume a correlation of 0.30, we will have 92% power.

## Concept: The Correlation Matrix

Assume that a subject has 5 measurements over time:  $Y_1, Y_2, Y_3, Y_4,$  and  $Y_5$

And, let  $\text{Corr}(Y_1, Y_2)$  represent the correlation between  $Y_1$  and  $Y_2$ , etc.

Then the **Correlation Matrix** represents all of the relationships between the five measurements:

|       | $Y_1$                   | $Y_2$                   | $Y_3$                   | $Y_4$                   | $Y_5$                   |
|-------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $Y_1$ | 1.0                     | $\text{Corr}(Y_1, Y_2)$ | $\text{Corr}(Y_1, Y_3)$ | $\text{Corr}(Y_1, Y_4)$ | $\text{Corr}(Y_1, Y_5)$ |
| $Y_2$ | $\text{Corr}(Y_2, Y_1)$ | 1.0                     | $\text{Corr}(Y_2, Y_3)$ | $\text{Corr}(Y_2, Y_4)$ | $\text{Corr}(Y_2, Y_5)$ |
| $Y_3$ | $\text{Corr}(Y_3, Y_1)$ | $\text{Corr}(Y_3, Y_2)$ | 1.0                     | $\text{Corr}(Y_3, Y_4)$ | $\text{Corr}(Y_3, Y_5)$ |
| $Y_4$ | $\text{Corr}(Y_4, Y_1)$ | $\text{Corr}(Y_4, Y_2)$ | $\text{Corr}(Y_4, Y_3)$ | 1.0                     | $\text{Corr}(Y_4, Y_5)$ |
| $Y_5$ | $\text{Corr}(Y_5, Y_1)$ | $\text{Corr}(Y_5, Y_2)$ | $\text{Corr}(Y_5, Y_3)$ | $\text{Corr}(Y_5, Y_4)$ | 1.0                     |

Note: We assume that every subject has the same correlation matrix

In Our Dataset: (Making no allowance for time trends)

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | .53      | .51       |
| 6 Months  | .53      | 1.0      | .69       |
| 12 Months | .51      | .69      | 1.0       |



## How Do We Estimate The Correlations?

We need to measure  $\text{Corr}(\varepsilon_{i1}, \varepsilon_{i2})$ , and  $\text{Corr}(\varepsilon_{i1}, \varepsilon_{i3})$ , and  $\text{Corr}(\varepsilon_{i2}, \varepsilon_{i3})$ , and ... etc.

Which means we need to have the  $\varepsilon$ 's, where

$$\varepsilon_{ij} = \text{CD4}_{ij} - \alpha - \beta \text{Time}_{ij}$$

Which means we need to have  $\alpha$  and  $\beta$ .

Which means we have to run the regression.

But to run the regression we need to know how to weight the data, so we need to know the correlations.

→ This leads to a tedious, iterative fitting algorithm which is not always successful.

## How Many Correlations Are We Talking About?

In our data, there are up to 3 measurements per subject, so there are only 3 unique correlation coefficients that need to be estimated.

However, if there were 15 serial measurements on a subject, there would be 105 unique correlation coefficients to be estimated.

Possible Simplifying Assumptions: We can help the process along by making some assumptions about the “form” of the correlation matrix. This will require the estimation of (many) fewer correlation coefficients, and lead to more stable answers concerning the regression parameters that are truly of interest.

### 1. Unstructured

|       | $Y_1$       | $Y_2$       | $Y_3$       | $Y_4$       | $Y_5$       |
|-------|-------------|-------------|-------------|-------------|-------------|
| $Y_1$ | 1.0         | $\rho_{12}$ | $\rho_{13}$ | $\rho_{14}$ | $\rho_{15}$ |
| $Y_2$ | $\rho_{21}$ | 1.0         | $\rho_{23}$ | $\rho_{24}$ | $\rho_{25}$ |
| $Y_3$ | $\rho_{31}$ | $\rho_{32}$ | 1.0         | $\rho_{34}$ | $\rho_{35}$ |
| $Y_4$ | $\rho_{41}$ | $\rho_{42}$ | $\rho_{43}$ | 1.0         | $\rho_{45}$ |
| $Y_5$ | $\rho_{51}$ | $\rho_{52}$ | $\rho_{53}$ | $\rho_{54}$ | 1.0         |

## Toeplitz

|       | $Y_1$    | $Y_2$    | $Y_3$    | $Y_4$    | $Y_5$    |
|-------|----------|----------|----------|----------|----------|
| $Y_1$ | 1.0      | $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_4$ |
| $Y_2$ | $\rho_1$ | 1.0      | $\rho_1$ | $\rho_2$ | $\rho_3$ |
| $Y_3$ | $\rho_2$ | $\rho_1$ | 1.0      | $\rho_1$ | $\rho_2$ |
| $Y_4$ | $\rho_3$ | $\rho_2$ | $\rho_1$ | 1.0      | $\rho_1$ |
| $Y_5$ | $\rho_4$ | $\rho_3$ | $\rho_2$ | $\rho_1$ | 1.0      |

## Autoregressive

|       | $Y_1$    | $Y_2$    | $Y_3$    | $Y_4$    | $Y_5$    |
|-------|----------|----------|----------|----------|----------|
| $Y_1$ | 1.0      | $\rho$   | $\rho^2$ | $\rho^3$ | $\rho^4$ |
| $Y_2$ | $\rho$   | 1.0      | $\rho$   | $\rho^2$ | $\rho^3$ |
| $Y_3$ | $\rho^2$ | $\rho$   | 1.0      | $\rho$   | $\rho^2$ |
| $Y_4$ | $\rho^3$ | $\rho^2$ | $\rho$   | 1.0      | $\rho$   |
| $Y_5$ | $\rho^4$ | $\rho^3$ | $\rho^2$ | $\rho$   | 1.0      |

## Exchangeable (Compound Symmetry)

|       | $Y_1$  | $Y_2$  | $Y_3$  | $Y_4$  | $Y_5$  |
|-------|--------|--------|--------|--------|--------|
| $Y_1$ | 1.0    | $\rho$ | $\rho$ | $\rho$ | $\rho$ |
| $Y_2$ | $\rho$ | 1.0    | $\rho$ | $\rho$ | $\rho$ |
| $Y_3$ | $\rho$ | $\rho$ | 1.0    | $\rho$ | $\rho$ |
| $Y_4$ | $\rho$ | $\rho$ | $\rho$ | 1.0    | $\rho$ |
| $Y_5$ | $\rho$ | $\rho$ | $\rho$ | $\rho$ | 1.0    |

## Data Analysis Results

| <u>Correlation Assumption</u> | <u>Monthly Change in CD4 Count (Slope)</u> | <u>Standard Error</u> | <u>P-value</u> |
|-------------------------------|--|-----------------------|----------------|
| 1. Independence               | 17.6                                       | 2.72                  | p<.001         |
| 2. Exchangeable               | 17.6                                       | 1.84                  | p<.001         |
| 3. Autoregressive             | 18.0                                       | 2.18                  | p<.001         |
| 4. Toeplitz                   | 17.8                                       | 1.98                  | p<.001         |
| 5. Unstructured               | 17.3                                       | 2.02                  | p<.001         |

## Estimated Correlation Matrices

### 1. Independence:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.0      | 0.0       |
| 6 Months  | 0.0      | 1.0      | 0.0       |
| 12 Months | 0.0      | 0.0      | 1.0       |

### 2. Exchangeable:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.56     | 0.56      |
| 6 Months  | 0.56     | 1.0      | 0.56      |
| 12 Months | 0.56     | 0.56     | 1.0       |

### 3. Autoregressive:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.62     | 0.38      |
| 6 Months  | 0.62     | 1.0      | 0.62      |
| 12 Months | 0.38     | 0.62     | 1.0       |

4. Toeplitz:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.61     | 0.49      |
| 6 Months  | 0.61     | 1.0      | 0.61      |
| 12 Months | 0.49     | 0.61     | 1.0       |

5. Unstructured:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.53     | 0.49      |
| 6 Months  | 0.53     | 1.0      | 0.68      |
| 12 Months | 0.49     | 0.68     | 1.0       |

## An Alternative Way To Accommodate Correlation **Random Coefficients Model (Growth Curve)**

$$CD4_{ij} = a_i + b_i \text{Time}_{ij} + \varepsilon_{ij}$$

Note: The slope and the intercept are now unique to each subject, rather than having one slope and one intercept which are representative of all subjects.

Assumptions: Across the subjects,  
The slopes come from a Normal distribution, centered at  $\beta$ , with variance  $\sigma_\beta^2$

These slopes are assumed to be independent from subject to subject. However, for a given subject, the same slope appears for each serial measurement and therefore there is correlation between serial measurements.

The intercepts come from a Normal distribution, centered at  $\alpha$ , with variance  $\sigma_\alpha^2$

These intercepts are assumed to be independent from subject to subject. However, for a given subject, the same intercept appears for each serial measurement and therefore there is correlation between serial measurements.

The error terms ( $\varepsilon$ 's) are assumed to be independent both within and between subjects



## Data Analysis Results

| <b><u>Model Assumptions</u></b> | <b><u>Monthly Change in CD4 Count (Slope)</u></b> | <b><u>Standard Error</u></b> | <b><u>P-value</u></b> |
|---------------------------------|---|------------------------------|-----------------------|
| 1. Random Intercepts            | 17.6  | 1.84                         | p<.001                |
| 2. Random Slopes                | 17.7  | 2.87                         | p<.001                |
| 3. Random Slopes and Intercepts | 17.8  | 1.99                         | p<.001                |

## Estimated Correlation Matrices

### 1. Random Intercepts:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.56     | 0.56      |
| 6 Months  | 0.56     | 1.0      | 0.56      |
| 12 Months | 0.56     | 0.56     | 1.0       |

### 2. Random Slopes:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.0      | 0.0       |
| 6 Months  | 0.0      | 1.0      | 0.31      |
| 12 Months | 0.0      | 0.31     | 1.0       |

### 3. Random Slopes and Intercepts:

|           | Baseline | 6 Months | 12 Months |
|-----------|----------|----------|-----------|
| Baseline  | 1.0      | 0.58     | 0.52      |
| 6 Months  | 0.58     | 1.0      | 0.64      |
| 12 Months | 0.52     | 0.64     | 1.0       |

| Obs | subject | cd4  | time | Age     | gender |
|-----|---------|------|------|---------|--------|
| 1   | JC-01*  | 1023 | 0    | 30.6438 | M      |
| 2   | JC-01*  | 1087 | 6    | 30.6438 | M      |
| 3   | JC-01*  | 651  | 12   | 30.6438 | M      |
| 4   | ND-02*  | 643  | 0    | 42.3507 | M      |
| 5   | ND-02*  | 561  | 6    | 42.3507 | M      |
| 6   | ND-02*  | 645  | 12   | 42.3507 | M      |
| 7   | SJ-03*  | 463  | 0    | 30.3315 | M      |
| 8   | SJ-03*  | 534  | 6    | 30.3315 | M      |
| 9   | SJ-03*  | 1019 | 12   | 30.3315 | M      |
| 10  | DK-04   | 652  | 0    | 34.9671 | M      |
| 11  | DK-04   | 777  | 6    | 34.9671 | M      |
| 12  | DK-04   | 682  | 12   | 34.9671 | M      |
| 13  | KM-05*  | 289  | 0    | 37.3753 | M      |
| 14  | KM-05*  | 608  | 6    | 37.3753 | M      |
| 15  | KM-05*  | 715  | 12   | 37.3753 | M      |
| 16  | KS-06*  | 551  | 0    | 36.1671 | M      |
| 17  | KS-06*  | 1085 | 6    | 36.1671 | M      |
| 18  | KS-06*  | 1037 | 12   | 36.1671 | M      |
| 19  | NF-07** | 131  | 0    | 22.8767 | F      |
| 20  | NF-07** | 548  | 6    | 22.8767 | F      |
| 21  | NF-07** | 496  | 12   | 22.8767 | F      |
| 22  | RM-08*  | 263  | 0    | 39.1918 | M      |
| 23  | RM-08*  | 575  | 6    | 39.1918 | M      |
| 24  | RM-08*  | 631  | 12   | 39.1918 | M      |
| 25  | JF-09   | 365  | 0    | 35.6795 | M      |
| 26  | JF-09   | 459  | 6    | 35.6795 | M      |
| 27  | JF-09   | 594  | 12   | 35.6795 | M      |
| 28  | AR-10   | 919  | 0    | 30.6959 | M      |
| 29  | AR-10   | 757  | 6    | 30.6959 | M      |
| 30  | AR-10   | 836  | 12   | 30.6959 | M      |
| 31  | JB-11   | 311  | 0    | 30.8438 | M      |
| 32  | JB-11   | 605  | 6    | 30.8438 | M      |
| 33  | JB-11   | 506  | 12   | 30.8438 | M      |
| 34  | ACE-12  | 311  | 0    | 62.3342 | M      |
| 35  | ACE-12  | 473  | 6    | 62.3342 | M      |
| 36  | ACE-12  | 408  | 12   | 62.3342 | M      |
| 37  | JFM-13  | 667  | 0    | 32.4027 | M      |
| 38  | JFM-13  | 814  | 6    | 32.4027 | M      |
| 39  | JFM-13  | 769  | 12   | 32.4027 | M      |
| 40  | GV-14   | 981  | 0    | 44.6466 | M      |
| 41  | GV-14   | 973  | 6    | 44.6466 | M      |
| 42  | GV-14   | 1276 | 12   | 44.6466 | M      |
| 43  | BS-15*  | 413  | 0    | 40.6986 | M      |
| 44  | BS-15*  | 527  | 6    | 40.6986 | M      |
| 45  | BS-15*  | 555  | 12   | 40.6986 | M      |
| 46  | MB-16   | 1093 | 0    | 34.3315 | M      |
| 47  | MB-16   | .    | 6    | 34.3315 | M      |
| 48  | MB-16   | 981  | 12   | 34.3315 | M      |
| 49  | SE-17   | 487  | 0    | 41.0329 | M      |
| 50  | SE-17   | 487  | 6    | 41.0329 | M      |
| 51  | SE-17   | 543  | 12   | 41.0329 | M      |
| 52  | RAS-18  | 303  | 0    | 38.4575 | M      |
| 53  | RAS-18  | 957  | 6    | 38.4575 | M      |
| 54  | RAS-18  | 406  | 12   | 38.4575 | M      |
| 55  | RC-19   | 443  | 0    | 55.7781 | M      |

```
proc mixed;  
  class subject;  
  model cd4 = time / solution;  
  repeated / type=ar(1) subject=subject rcorr;
```

```
proc mixed;  
  class subject;  
  model cd4 = time age / solution;  
  repeated / type=toep subject=subject rcorr;
```

```
proc mixed;  
  class subject gender;  
  model cd4 = time age gender / solution;  
  repeated / type=uns subject=subject rcorr;
```

The SAS System            09:46 Tuesday, May 8, 2012 5  
 The Mixed Procedure  
 Model Information

Data Set            WORK.LONGIDTUDINAL  
 Dependent Variable    cd4  
 Covariance Structure    Autoregressive  
 Subject Effect        subject  
 Estimation Method     REML  
 Residual Variance Method   Profile  
 Fixed Effects SE Method   Model-Based  
 Degrees of Freedom Method   Between-Within

Class Level Information

| Class   | Levels | Values   |
|---------|--------|--|
| subject | 103    | ACE-12 AES-47 ALR-43 AP-22*<br>AR-10 ASB-46* AV-35 BAB-25<br>BBB-125 BDD-56 BKC-32 BS-15*<br>BSB-31 CCG-45 CP-52 CWM-102<br>DD-58 DIC-10 7 DJ5-68 DK-04<br>DM-116 DMD-42 DRB-23 DT-90<br>DWS-85 EAS-80 ENS-41 EO-20<br>ESJ-27 FCHC 0135-78 FCHC<br>1026-64 FCHC 1029-67 FCHC<br>1031-77 FCHC 1033-73 FCHC<br>1045-99 FCHC 1049-111 FCHC<br>1051-112 FCHC1009-44<br>FCHC1010-40* FCHC1013-48 FD-96<br>FS-114 GAD-59 GI-88 GV-14<br>IC-51 JB-11 JC-01* JDB-74<br>JF-09 JFM-13 JGN-100 JH-33<br>JLS-70 JMV-63 JRP-54 JXV-84 KH<br>-105 KJS-66* KM-05* KS-06*<br>LFD-39 MB-16 MCW-29 MDL-81<br>ME-75 MJH-72 MJK-95 ML-37<br>MLD-86 MLL-61* MRB-62 MTF-76<br>ND-02* NEC-119 NF-07** NJB-28<br>NTL-106 PCF-26 PS-87 RAB-117<br>RAS-18 RC-19 RDS-103 REM-101<br>RJ-94 RJS-30 RLO-113 RM-08*<br>RSN-91 SE-17 SG-34 SI-03*<br>SO-65 SRC-21 TD-36 TNT-55<br>WB-24 WF-104 WJO-82 WPC-69<br>WRC-60 WSB-118 |

Dimensions

|                       |     |
|-----------------------|-----|
| Covariance Parameters | 2   |
| Columns in X          | 2   |
| Columns in Z          | 0   |
| Subjects              | 103 |
| Max Obs Per Subject   | 3   |

|                                 |     |
|---------------------------------|-----|
| Number of Observations          |     |
| Number of Observations Read     | 309 |
| Number of Observations Used     | 267 |
| Number of Observations Not Used | 42  |

Iteration History

| Iteration | Evaluations | -2 Res Log Like | Criterion  |
|-----------|-------------|-----------------|------------|
| 0         | 1           | 3620.75413134   |            |
| 1         | 2           | 3547.30774003   | 0.00000666 |
| 2         | 1           | 3547.29745747   | 0.00000000 |

Convergence criteria met.

Estimated R Correlation Matrix  
for subject ACE-12

| Row | Col1   | Col2   | Col3   |
|-----|--------|--------|--------|
| 1   | 1.0000 | 0.6186 | 0.3826 |
| 2   | 0.6186 | 1.0000 | 0.6186 |
| 3   | 0.3826 | 0.6186 | 1.0000 |

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate |
|----------|---------|----------|
| AR(1)    | subject | 0.6186   |
| Residual |         | 48563    |

Fit Statistics

|                          |        |
|--------------------------|--------|
| -2 Res Log Likelihood    | 3547.3 |
| AIC (smaller is better)  | 3551.3 |
| AICC (smaller is better) | 3551.3 |
| BIC (smaller is better)  | 3556.6 |

Null Model Likelihood Ratio Test

| DF | Chi-Square | Pr > ChiSq |
|----|------------|------------|
| 1  | 73.46      | <.0001     |

Solution for Fixed Effects

| Effect    | Standard |         | DF  | t Value | Pr >  t |
|-----------|----------|---------|-----|---------|---------|
|           | Estimate | Error   |     |         |         |
| Intercept | 491.27   | 21.8234 | 100 | 22.51   | <.0001  |
| time      | 18.0256  | 2.1757  | 165 | 8.29    | <.0001  |

## The Mixed Procedure

## Model Information

Data Set WORK.LONGITUDINAL  
 Dependent Variable cd4  
 Covariance Structure Toeplitz  
 Subject Effect subject  
 Estimation Method REML  
 Residual Variance Method Profile  
 Fixed Effects SE Method Model-Based  
 Degrees of Freedom Method Between-Within

## Dimensions

Covariance Parameters 3  
 Columns in X 3  
 Columns in Z 0  
 Subjects 103  
 Max Obs Per Subject 3

## Class Level Information

| Class   | Levels | Values  |
|---------|--------|---|
| subject | 103    | ACE-12 AES-47 ALR-43 AP-22*<br>AR-10 ASB-46* AV-35 BAB-25<br>BBB-125 BDD-56 BKC-32 BS-15*<br>BSB-31 CCG-45 CP-52 CWM-102<br>DD-58 DIC-107 DJS-68 DK-04<br>DM-116 DMD-42 DRB-23 DT-90<br>DWS-85 EAS-80 ENS-41 EO-20<br>ESJ-27 FCHC 0135-78 FCHC<br>1026-64 FCHC 1029-67 FCHC<br>1031-77 FCHC 1033-73 FCHC<br>1045-99 FCHC 1049-111 FCHC<br>1051-112 FCHC1009-44<br>FCHC1010-40* FCHC1013-48 FD-96<br>FS-114 GAD-59 GI-88 GV-14<br>IC-51 JB-11 JC-01* JDB-74<br>JF-09 JFM-13 JGN-100 JH-33<br>JLS-70 JMV-63 JRP-54 JXV-84 KH<br>-105 KJS-66* KM-05* KS-06*<br>LFD-39 MB-16 MCW-29 MDL-81<br>ME-75 MJH-72 MJK-95 ML-37<br>MLD-86 MLL-61* MRB-62 MTF-76<br>ND-02* NEC-119 NF-07** NJB-28<br>NTL-106 PCF-26 PS-87 RAB-117<br>RAS-18 RC-19 RDS-103 REM-101<br>RJ-94 RJS-30 RLO-113 RM-08*<br>RSN-91 SE-17 SG-34 SJ-03*<br>SO-65 SRC-21 TD-36 TNT-55<br>WB-24 WF-104 WJO-82 WPC-69<br>WRC-60 WSB-118 |

Number of Observations  
 Number of Observations Read 309  
 Number of Observations Used 267  
 Number of Observations Not Used 42

Iteration History

| Iteration | Evaluations | -2 Res Log Like | Criterion  |
|-----------|-------------|-----------------|------------|
| 0         | 1           | 3617.79552161   |            |
| 1         | 2           | 3541.26198379   | 0.00014941 |
| 2         | 1           | 3541.01796083   | 0.00000206 |
| 3         | 1           | 3541.01478757   | 0.00000000 |

Fit Statistics

|                          |        |
|--------------------------|--------|
| -2 Res Log Likelihood    | 3541.0 |
| AIC (smaller is better)  | 3547.0 |
| AICC (smaller is better) | 3547.1 |
| BIC (smaller is better)  | 3554.9 |

Convergence criteria met.

Estimated R Correlation Matrix  
for subject ACE-12

| Row | Col1   | Col2   | Col3   |
|-----|--------|--------|--------|
| 1   | 1.0000 | 0.6122 | 0.4935 |
| 2   | 0.6122 | 1.0000 | 0.6122 |
| 3   | 0.4935 | 0.6122 | 1.0000 |

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate |
|----------|---------|----------|
| TOEP(2)  | subject | 29651    |
| TOEP(3)  | subject | 23902    |
| Residual |         | 48433    |

Null Model Likelihood Ratio Test

| DF | Chi-Square | Pr > ChiSq |
|----|------------|------------|
| 2  | 76.78      | <.0001     |

Solution for Fixed Effects

| Effect    | Standard |         | DF  | t Value | Pr >  t |
|-----------|----------|---------|-----|---------|---------|
|           | Estimate | Error   |     |         |         |
| Intercept | 474.41   | 83.2257 | 99  | 5.70    | <.0001  |
| time      | 17.8324  | 1.9851  | 165 | 8.98    | <.0001  |
| Age       | 0.5355   | 2.1644  | 99  | 0.25    | 0.8051  |



The Mixed Procedure

Model Information

Data Set WORK.LONGIDTUDINAL  
 Dependent Variable cd4  
 Covariance Structure Unstructured  
 Subject Effect subject  
 Estimation Method REML  
 Residual Variance Method None  
 Fixed Effects SE Method Model-Based  
 Degrees of Freedom Method Between-Within

Class Level Information

| Class   | Levels | Values  |
|---------|--------|---|
| subject | 103    | ACE-12 AES-47 ALR-43 AP-22*<br>AR-10 ASB-46* AV-35 BAB-25<br>BBB-125 BDD-56 BKC-32 BS-15*<br>BSB-31 CCG-45 CP-52 CWM-102<br>DD-58 DIC-107 DJS-68 DK-04<br>DM-116 DMD-42 DRB-23 DT-90<br>DWS-85 EAS-80 ENS-41 EO-20<br>ESJ-27 FCHC 0135-78 FCHC<br>1026-64 FCHC 1029-67 FCHC<br>1031-77 FCHC 1033-73 FCHC<br>1045-99 FCHC 1049-111 FCHC<br>1051-112 FCHC1009-44<br>FCHC1010-40* FCHC1013-48 FD-96<br>FS-114 GAD-59 GI-88 GV-14<br>IC-51 JB-11 JC-01* JDB-74<br>JF-09 JFM-13 JGN-100 JH-33<br>JLS-70 JMV-63 JRP-54 JXV-84 KH<br>-105 KJS-66* KM-05* KS-06*<br>LFD-39 MB-16 MCW-29 MDL-81<br>ME-75 MJH-72 MJK-95 ML-37<br>MLD-86 MLL-61* MRB-62 MTF-76<br>ND-02* NEC-119 NF-07** NJB-28<br>NTL-106 PCF-26 PS-87 RAB-117<br>RAS-18 RC-19 RDS-103 REM-101<br>RJ-94 RJS-30 RLO-113 RM-08*<br>RSN-91 SE-17 SG-34 SJ-03*<br>SO-65 SRC-21 TD-36 TNT-55<br>WB-24 WF-104 WJO-82 WPC-69<br>WRC-60 WSB-118 |
|         | 2      | F M   |

Dimensions

Covariance Parameters 6  
 Columns in X 5  
 Columns in Z 0  
 Subjects 103  
 Max Obs Per Subject 3  
 Number of Observations  
 Number of Observations Read 309  
 Number of Observations Used 267  
 Number of Observations Not Used 42

Iteration History

| Iteration | Evaluations | -2 Res Log Like | Criterion  |
|-----------|-------------|-----------------|------------|
| 0         | 1           | 3607.83497574   |            |
| 1         | 2           | 3525.89724948   | 0.00000538 |
| 2         | 1           | 3525.88893623   | 0.00000000 |

Convergence criteria met.

Estimated R Correlation Matrix  
for subject ACE-12

| Row | Col1   | Col2   | Col3   |
|-----|--------|--------|--------|
| 1   | 1.0000 | 0.5352 | 0.5013 |
| 2   | 0.5352 | 1.0000 | 0.6831 |
| 3   | 0.5013 | 0.6831 | 1.0000 |

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate |
|----------|---------|----------|
| UN(1,1)  | subject | 48169    |
| UN(2,1)  | subject | 24333    |
| UN(2,2)  | subject | 42911    |
| UN(3,1)  | subject | 25606    |
| UN(3,2)  | subject | 32930    |
| UN(3,3)  | subject | 54158    |

Fit Statistics

|                          |        |
|--------------------------|--------|
| -2 Res Log Likelihood    | 3525.9 |
| AIC (smaller is better)  | 3537.9 |
| AICC (smaller is better) | 3538.2 |
| BIC (smaller is better)  | 3553.7 |

Null Model Likelihood Ratio Test

| DF | Chi-Square | Pr > ChiSq |
|----|------------|------------|
| 5  | 81.95      | <.0001     |

Solution for Fixed Effects  
Standard

| Effect    | gender | Estimate | Error   | DF | t Value | Pr >  t |
|-----------|--------|----------|---------|----|---------|---------|
| Intercept |        | 482.96   | 84.5738 | 98 | 5.71    | <.0001  |
| time      |        | 17.2929  | 2.0208  | 98 | 8.56    | <.0001  |
| Age       |        | 0.4622   | 2.1803  | 98 | 0.21    | 0.8326  |
| gender    | F      | 18.9139  | 79.1864 | 98 | 0.24    | 0.8117  |
| gender    | M      | 0        | .       | .  | .       | .       |